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Original Article

Probabilistic Modeling and Structural Reliability based Monte Carlo Simulation: A Case Study

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Abstract *- The paper presents the Monte Carlo method and its applications. According to several studies, the method has been found to be a powerful, simple tool and has been widely utilized for a broad range of approaches to civil engineering problems. The paper also aims to explore the method for reliability analysis and probabilistic modeling. It provides an understanding through two practical cases of study, with a discussion of its fundamental advantages and drawbacks. The first application focuses on modeling the strength characteristics of a reinforced concrete cross-section, including bending moment and shear resistances. In contrast, the second case concentrates on the reliability analysis of a flanged bridge beam in accordance with the prescriptions of the European standard of reinforced concrete design, EUROCODE 2. The variations and uncertainties in geometrical parameters and material properties are modeled by deterministic and random variables using site measurements and the Joint Committee of Structural Safety (JCSS) Probabilistic Code, as well as the proposed models in the literature. This study offers significant contributions to the field of reliability, with a specific focus on structural engineering, and paves the way for further advancements in the use of artificial intelligence techniques, providing valuable insight for researchers and civil engineering practitioners.*

Keywords *- Monte Carlo simulation, Probability distribution, Structural reliability, FORM, Reinforced concrete, Bridges.*

1. Introduction

The simulation of Monte Carlo is a statistical approach that employs random sampling and simulation to solve many scientific problems [1]. It has been widely adopted in numerous scientific and technological fields such as engineering, finance, mathematics, physics, medicine, project management, biology, computer sciences, and many others [2] [3] [4]. The application of this method covers all areas where the implementation of scientific methods faces challenges. Within this context, there are two main domains where the Monte Carlo method can be effectively implemented: deterministic and stochastic problems. The first includes problems such as surface calculations, numerical integrations, and solving complex differential equations. The second is described by random processes such as the production process, risk management, safety assessment, and reliability problems. In regards to civil engineering, a substantial amount of literature has been published on the application of Monte Carlo simulation for probabilistic modeling and structural reliability analysis. These studies are further discussed in Section 2. A structural system should satisfy specific functionalities under well-defined safety conditions. Such conditions must be accounted for during the structural design phase to figure out all expected types of loads according to the design standard. Civil engineering experiments and construction processes, like project planning, structural design, and building stages, involve a certain degree of uncertainty. In fact, these uncertainties, according to [1], can result from two broad categories of causes: natural origins and human causes. The first type results from the unpredictability of natural actions and the randomness of their loading values; most cases are environmental actions (e.g. earthquakes, exceptional weather conditions like intense wind) and operating loads (e.g. traffic loads). The second type involves intentional or unintentional deviations from an optimal design. For example, uncertainties through the conception and design due to approximations, calculation errors, lack of knowledge, and communication problems. Similarly, unpredictability occurs through inadequate formulation of construction materials on the construction site (e.g. high-strength concrete), practical execution difficulties, as well as the fact that maintenance operations may overload the structure.

In the presence of all those uncertainties, it can occur that the structural system operates outside its safety domain and exceeds its load-bearing capacity. In such cases, the structural system fails. Therefore, the ability to fulfill all safety demands under such uncertainties is called reliability. In this case, structural reliability methods aim to calculate a probability of failure, as follows, a reliability index is estimated. For a detailed review of reliability methods, see [1] and [5].

The basement methodology of structural reliability is a probabilistic approach based on the concept of performance functions to describe the failure conditions by reliability methods, which are mainly based on two types of strategies which are approximation and simulation [6]. Approximation methods are based on expanding the performance functions at a reference point (e.g. Taylor expansion), which can be very efficient but tends to become unreliable in the case of complex performance functions. The most commonly used approximation methods are FORM [1]. Simulation techniques called sampling methods are commonly based on Monte Carlo simulation, a technique that generates numerical results without actually doing any physical testing [1].

Indeed, involving structural engineering, Monte Carlo simulation covers a variety of studies in a wide range of disciplines. For instance, relating to the economy, the objective in [2] was to evaluate the financial feasibility of a manufacturing sector in Sao Paulo, Brazil, through a Monte Carlo simulation.

In the medical field, [3] used the method to determine the quality-adjusted life-year impact resulting from the implementation of common preventive health interventions on individuals' health. For project management, the main idea in [4] is to examine the utilization of the simulation method for planning management. The authors conducted a literature review focusing on the application of simulation in quantitative risk analysis and proposed potential improvements related to project schedule management. Hence, in the context of research studies related to the civil engineering and construction laboratory of CEDOC-EMI, this paper reflects important issues for advanced structural engineering in Morocco, especially those about safety and reliability.

In addition, this study builds upon presenting a literature review of Monte Carlo simulation in previous research with diverse applications in reliability engineering and demonstrating its practical application for probabilistic modeling of the resistance characteristics related to a reinforced concrete cross-section and structural reliability, considering a bridge girder as a case study. The research gap of the study is to present a novel contribution perspective for advancing the use of probabilistic sampling methods in structural reliability, providing valuable insight for researchers and civil engineering practitioners.

2. Literature Review

Due to the complexity of civil engineering structures, it is seldom possible to identify analytical solutions using approximation techniques [7]. As a result, the Monte Carlo method is widely preferred and has been the subject of many diverse studies for probabilistic modeling and reliability in civil engineering. The focus of these studies varies widely. The authors in [8] used the simulation technique to calculate the reliability index of a reinforced concrete box girder with a probabilistic approach. For the considered application, [8] collected the material's properties and statistical parameters from experimental data. The authors performed an analysis to figure out the bridge collapse load and the load multiplier to calculate a reliability index. The sensitivity check ensures that the number of simulations takes a high amount of time. Thus, variance reduction techniques such as important sampling are preferred, as recommended in [7], which provides adaptive importance sampling based on the Monte Carlo method and reduces the computations required.

The suggested algorithm, according to the authors, is very efficient as a developed method and presents a lot of capabilities in terms of computing cost. At the same time, multiple examples from the literature are studied. The principal aim in [9] was to establish a 3D model for a prestressed concrete beam and to calculate the failure probability using Monte Carlo simulation. The authors adopt the finite element software ANSYS for structural analysis to define failure modes. Hence, the limit state functions related to stiffness, strength, and durability were listed. [10] research has provided a reliability analysis for the response's wave transmission at peak frequencies for a deteriorated beam, considering the influence of uncertainty in parameters. For this purpose, the authors suggest using the FORM and response surface technique to assess the probability of failure. Then, the results were compared with the simulation of Monte Carlo. In the machine learning context, [11] considered a method that links neural networks with Monte Carlo simulation to enhance the computing operation time cost. The authors presented application examples on a test function, a single steel frame, and a six-bar simply supported truss system. The results conclude a reduction in computation time, and the fact of conjoining neural networks with the two techniques creates a strong potential for structural reliability. The authors in [12] focused on the reliability assessment of bridges subjected to time-dependent deterioration.

The article presented a methodology to compute the load effect on structure lifetime due to existing traffic and resistance loss by incorporating the corrosion rate due to chloride ingression. [13] exanimated the impact of dynamic vehicle wheel loads and the influence of impact factor on the fatigue function of reinforced concrete slabs. As a case study, a simple bridge span with a composite steel plate girder is considered. The authors created a 3D traffic-induced dynamic analysis to simulate the impact coefficient using Monte Carlo simulation. As well, a reliability evaluation was conducted on the experimental results to assess the probability of failure in fatigue limit state mode. For geotechnical engineering, [14] focused on a slope stability analysis using a probabilistic approach based on the simplified Bishop method and Monte Carlo simulation.

The study adopted cohesion and the soil's friction angle as random parameters. The proposed approach is implemented as a calculation program that allows the identification of the critical circle and provides a safety factor, probability of failure, and reliability index. According to [14], the resulting index contains more information than the safety factor because it incorporates the spatial variability of soil properties, thereby implicitly using all information from geotechnical tests.

The research of [15] pointed to Monte Carlo simulation for quantifying the impact of time-related loads like corrosion on the durability of reinforced concrete structures. The study focuses on a reinforced concrete slab bridge, using Monte Carlo simulation to derive a failure rate for the deterioration of reinforcement steel caused by atmospheric marine exposure. The results found that the impact of those environmental actions causes important long-term damage with a decrease in structural safety.

The authors noted that preliminary models of corrosion developed should be refined by including various factors that may influence corrosion initiation and propagation (e.g. chloride surface concentration, diffusion coefficient, concrete carbonation, chloride-induced corrosion, etc.). The overview of the literature conducted in this section serves as a perspective presentation of probabilistic sampling technique implementation based on the Monte Carlo method for multiple applications in civil engineering.

3. Monte Carlo Simulation

Generally, the idea is to use random sampling to simulate the possible outcomes of a given problem. Stanislaw Ulam and John von Neumann first introduced the Monte Carlo simulation in 1949 [16], and its name comes from the Monte Carlo casino in Monaco, which is famous for its use of randomness and probability in chance games. The method can be used to estimate the probability of outcomes, to optimize the performance of a system, or to determine the sensitivity of a system to changes in its inputs. According to [17], the Monte Carlo simulation depends on representing each parameter of the problem by its probabilistic distribution, mean, and standard deviation.

The steps in Table 1 describe the computing procedure, as stated in [18]. The flowchart in Figure 1 is given as an example, where five main steps are required for the simulation process. A reliability problem is formulated by a performance function known as a limit state function $G(X_i)$, where the random variables of the problem are $X_i = (X_0, X_1, X_2, ..., X_n)$.

Table 1. Steps of Monte Carlo simulation [18]

Step	Description
1	Defining the performance function of the outcome problem is fixed in terms of all stochastic and deterministic variables.
2	Statistical parameters for all random variables are quantified in terms of distribution.
3	A sampling set for each random variable is generated from its probabilistic distribution.
4	Simulations are conducted to evaluate the performance function using the generated sampling set.
5	Probabilistic information after N realizations is extracted to estimate the outcome of the problem.
6	The accuracy of the simulation is checked by increasing the samples.

The violation of the safety criterion is defined by $G(X_i) \leq 0$, and the failure's probability is expressed as given in (1).

$$P_f = P[G(X_i) \leq 0] = \int_{G(X_i)\leq 0} f_{X_i}(x_i)dx_i \quad (1)$$

Where $f_{X_i}(x_i)$ is the joint density function.

Monte Carlo simulation allows the calculation of the probability of failure [11], given by the following expression (2).

$$P_f = \frac{1}{N}\sum_{i}^{n} N(X_i) \quad (2)$$

$N(X_i)$ is a function defined as follows:

$$N(X_i) = \begin{cases} 1 \; if \; G(X_i) \leq 0 \\ 0 \; if \; G(X_i) > 0 \end{cases} \quad (3)$$

N independent sets of values are related to the distribution of each random variable [7]. The probability of failure is obtained then by (4).

$$P_f = \frac{N_F}{N} \quad (4)$$

N_F is the number of failure cases. Therefore, the reliability index can be expressed as given in (5).

$$\beta = -\phi^{-1}(P_f) \; with \; \phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2}dt \quad (5)$$

4. Probabilistic Models

The development of probabilistic models for basic random variables is considered the main task of reliability analysis. [18] presents the essential steps in developing probabilistic models for uncertainties. They show that histograms are used to describe the characteristics of uncertainty, and these identify the probability density function. [19] divides probabilistic modeling into the evaluation and statistical quantification of available data, selection of the distribution function, estimation of distribution parameters, for example, and model verification.

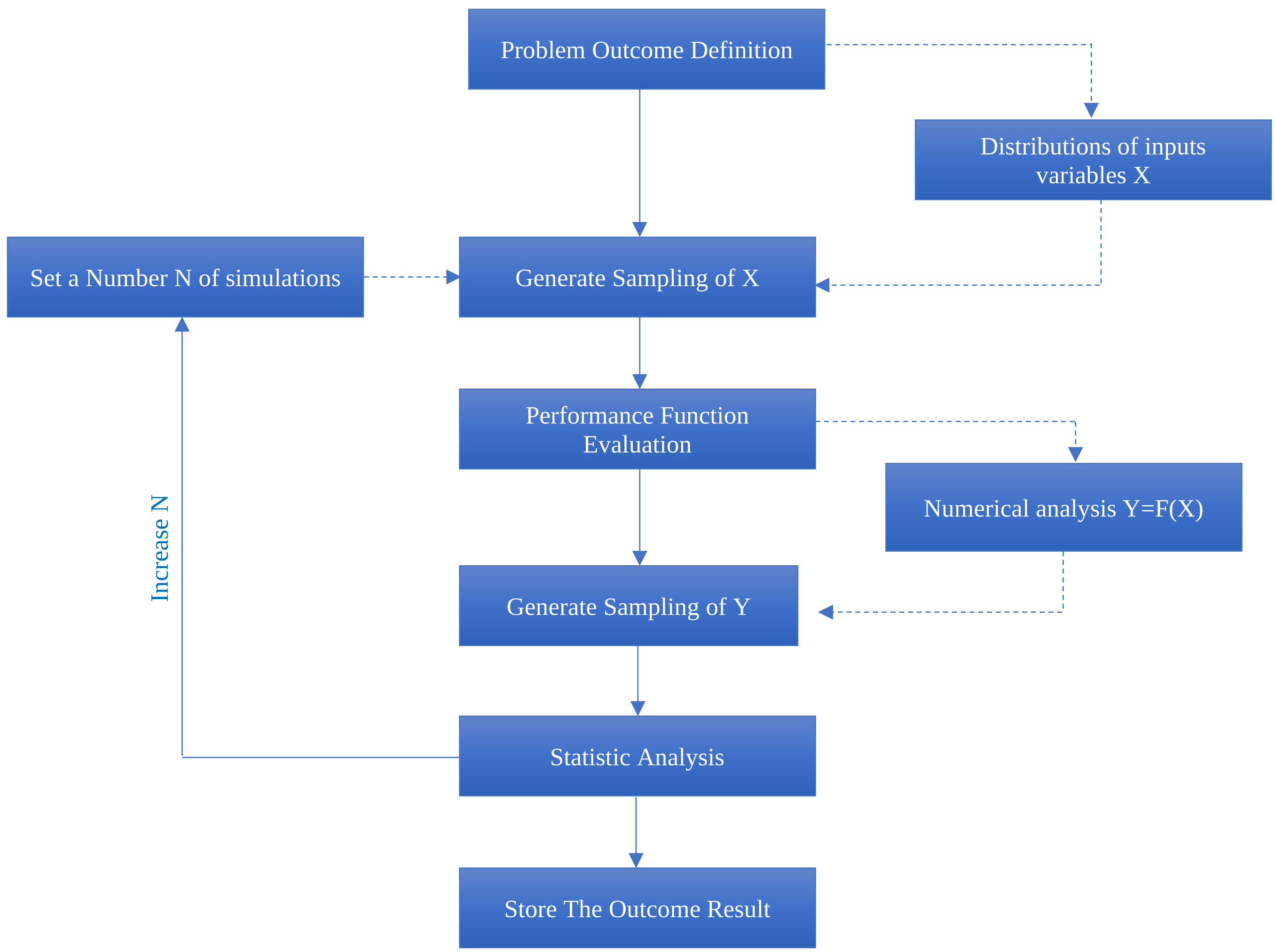

Fig 1. Methodology's flowchart

Certain parameters, known as statistics, need to be estimated in order to describe the probability density function. The estimation of these parameters, the mean and standard deviation, is the main component of uncertainty analysis. Consequently, the randomness of the performance function can be quantified using these statistics. The JCSS code [20] and the literature present various probabilistic models for reinforced concrete and steel material properties.

4.1. Concrete Properties

The mechanical properties of concrete vary considerably for three different reasons. The first is variation in the properties of the materials used to form concrete, such as the properties of cement and fine and coarse aggregates. The second is manufacturing, which includes concrete composition (variation in components, water content, cement, and aggregates) and execution methods, which include mixing, transport, temperature, etc. The last reason concerns testing, which includes, for example, sampling, specimen preparation, curing, and testing procedures and equipment. Various probabilistic models have been developed in the literature for the mechanical properties of concrete based on observations and tests. Several probabilistic models for the compressive and tensile mechanical properties of concrete have been proposed in the literature, notably by [21] [22] [23] and [24], as presented in Tables 2 and 3.

Table 2. Probabilistic models of concrete compressive strength

Parameter	PDF	Mean	COV	Std	Ref
f_c	Log	$0.675f_{cm} + 7.7$	0.12-0.20	-	[21]
f_c	Log	$f_{cm} + 7.5$	-	6	[22]
f_c	Norm	$1.03f_{cm}$	0.18	-	[23]
$f_c(Site)$	Log	f_{cm}	0.12	-	[24]
$f_c(Plant)$	Log	f_{cm}	0.09	-	[24]

Table 3. Probabilistic models of concrete tensile strength

Parameter	PDF	Mean	COV	Ref
f_t	Norm	$0.69\sqrt{f_{cm}}$	0.2	[21]
f_t	Norm	$0.35\sqrt{f_{cm}}$	0.13	[21]
f_t	Norm	$0.3f_{cm}^{\frac{2}{3}}$	0.2	[22]

Table 4. Statistical parameters of reinforcement steel properties

Parameter	PDF	Mean	COV	Std	Ref
f_y	Normal	$S_n + 2\sigma$	-	30	[20]
f_y	Béta	S_n	0.1	-	21]
E_s	Normal	$1.005E_{sn}$	0.033	-	[21]

4.2. Reinforcement Steel Properties

The properties of reinforcing and prestressing steel in structural engineering are studied in terms of mechanical properties, mainly yield strength and modulus of elasticity. The construction environment and variations in steel composition can affect material properties. In the literature, several probabilistic models for reinforcing and prestressing steel have been developed. Some of the models commonly used in the literature have been summarized in Table 4. Where S_n represents the nominal value corresponding to the steel grade, e.g. S300 and S400. $E_{sn}$is the nominal value of the modulus of elasticity.

5. Results

5.1. Case Study: Probabilistic Modeling of strength properties

5.1.1. Structural Design Limits

In order to provide an insight into Monte Carlo simulation, a reinforced concrete beam is examined. The beam, with a span length of 5.53 meters, is simply supported, and it is designed for bending according to the EUROCODE 2 [25]. The concrete has a nominal compressive strength of 25 MPa, while the steel reinforcement has a nominal yield stress of 500 MPa. The beam has a nominal depth of 0.45 meters and a nominal width of 0.25 meters. Additionally, there are nine reinforcement bars, each with a 12 mm diameter, and they are embedded, as illustrated in Figure 2. The variation of width and height along the beam are presented in Figure 3.

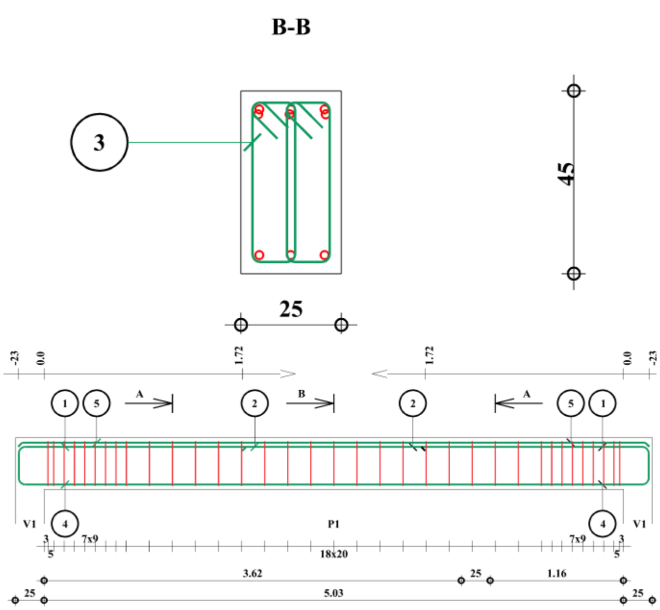

Fig. 2 Reinforcement details

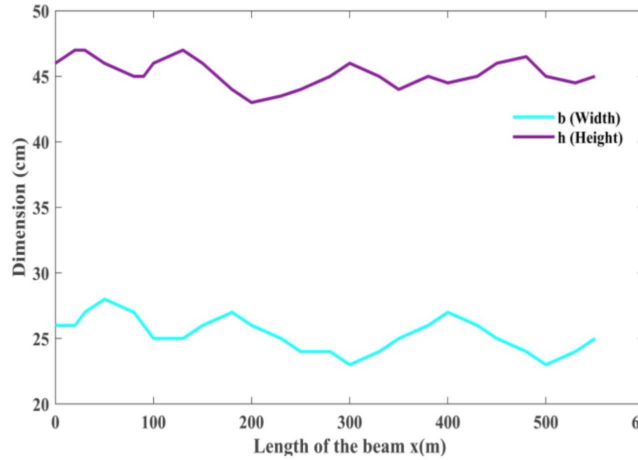

Fig. 3 The variation of dimensions along the beam

The objective is to estimate the extreme values and the probability distribution of resistance properties for an RC beam section using Monte Carlo simulation. Concrete's compressive strength and steel yield stress are treated, respectively, as lognormal and normal variables. The interval values for these variables are based on their mean and coefficient of variation, which have been adopted from [20] and [25] and based on the following formulas (6) and (7).

$$Xnom(1 + cov(X))_{max} \tag{6}$$

$$Xnom(1 - cov(X))_{min} \tag{7}$$

A normal distribution describes the uncertainty of geometry parameters. The random variables are presented in Table 5. According to EUROCODE 2 [25], bending and shear resistances at the ultimate limit state of a reinforced concrete rectangular section are respectively equal to (8) and (9).

$$M_R = 0.87A_s f_y \left(d - \frac{0.87A_s f_y}{1.134bf_{c28}}\right) \tag{8}$$

$$V_R = 0.124bdf_{c28}\left(1 - \frac{f_{c28}}{250}\right) \tag{9}$$

With:
- f_{c28} : 28th day Concrete compressive strength
- f_y : Steel yield stress
- A_s : Reinforcement cross-section
- b : Width of the beam
- d : Effective height of the beam

5.1.2. Results

The number of iterations is set at 50,000, using the described input random variables. The numerical implementation of the simulation is based on a random sampling of numbers. Random variables were introduced using the Microsoft Excel formula [RAND.BETWEEN (Max Value; Min Value)] in the English version or [ALEA.ENTRE.BORNES (Valeur Max; Valeur Min)] in the French version. The simulated probabilistic values of both strength properties are given in Tables 6 and 7.

The graphs shown in Figure 4 represent the resulting probability distribution and cumulative distribution functions for bending moment and shear resistances, respectively. Then, the estimated statistical parameters are given in Table 8. These results suggest that the bending resistance adheres to a normal probability distribution. It has a mean value of 146.27 kN.m with a standard deviation of 11.25. The estimated interval for extreme values falls within the range of [129.27, 163.27].

Furthermore, the shear resistance is characterized by a lognormal probability distribution. Its mean is 284.66 kN.m, with a standard deviation of 50.49. These findings support the key advantage of the Monte Carlo method in modeling the probability distribution of an outcome and quantifying its statistical parameters.

The results of the 50,000 simulations indicate that the frequency of obtaining a value of moment resistance within the interval [145.27; 147.27] is equal to 12.25%. The corresponding cumulative frequency of getting a value equal to or less than 146.27 kN.m is approximately 65%. Similarly, the probability of obtaining a shear resistance's value within the interval [284.66; 294.66] is equal to 11%, and the frequency of obtaining a value equal to or less than 284.66 kN.m is approximately 63%.

Table 5. Statistical data of the problem's variables

Var.	Unit	PDF	Nominal	Max	Min
f_y	MPa	Log	500	530	470
f_{c28}	MPa	Log	25	29.14	20.86
h	m	Norm	0.45	0.47	0.43
b	m	Norm	0.25	0.27	0.23
e	m	Norm	0.025	0.03	0.02
d	m	Norm	0.425	0.45	0.40
A_S	m²	Det	10.18	-	-

Table 6. Monte Carlo simulation for bending moment resistance

B-M Resistance		Mean	PDF	CDF
129,27	131,27	130,27	0,21%	0,21%
131,27	133,27	132,27	0,67%	0,89%
133,27	135,27	134,27	1,70%	2,59%
135,27	137,27	136,27	3,25%	5,84%
137,27	139,27	138,27	5,53%	11,37%
139,27	141,27	140,27	8,07%	19,44%
141,27	143,27	142,27	10,12%	29,56%
143,27	145,27	144,27	11,94%	41,49%
145,27	147,27	146,27	12,25%	53,75%
147,27	149,27	148,27	12,06%	65,80%
149,27	151,27	150,27	10,39%	76,19%
151,27	153,27	152,27	8,81%	85,00%
153,27	155,27	154,27	6,46%	91,46%
155,27	157,27	156,27	4,36%	95,82%
157,27	159,27	158,27	2,52%	98,34%
159,27	161,27	160,27	1,20%	99,54%
161,27	163,27	162,27	0,43%	99,97%
163,27	165,27	164,27	0,03%	100,00%

Table 7. Monte Carlo simulation results for shear resistance

Shear Resistance		Mean	PDF	CDF
204,66	214,66	209,66	3,25%	3,25%
214,66	224,66	219,66	4,02%	7,27%
224,66	234,66	229,66	5,42%	12,69%
234,66	244,66	239,66	7,50%	20,19%
244,66	254,66	249,66	9,47%	29,66%
254,66	264,66	259,66	11,21%	40,87%
264,66	274,66	269,66	10,98%	51,85%
274,66	284,66	279,66	10,75%	62,60%
284,66	294,66	289,66	9,47%	72,07%
294,66	304,66	299,66	7,23%	79,30%
304,66	314,66	309,66	7,11%	86,41%
314,66	324,66	319,66	5,47%	91,88%
324,66	334,66	329,66	3,25%	95,13%
334,66	344,66	339,66	2,43%	97,56%
344,66	354,66	349,66	1,70%	99,26%
354,66	364,66	359,66	0,74%	100,00%

Table 8. Statistical results of Monte Carlo Simulation

Prop.	Unit	Min	Max	Mean	Std
M_R	kN.m	129.27	163.27	146.27	11.25
V_R	kN	204.66	364.66	284.66	50.49

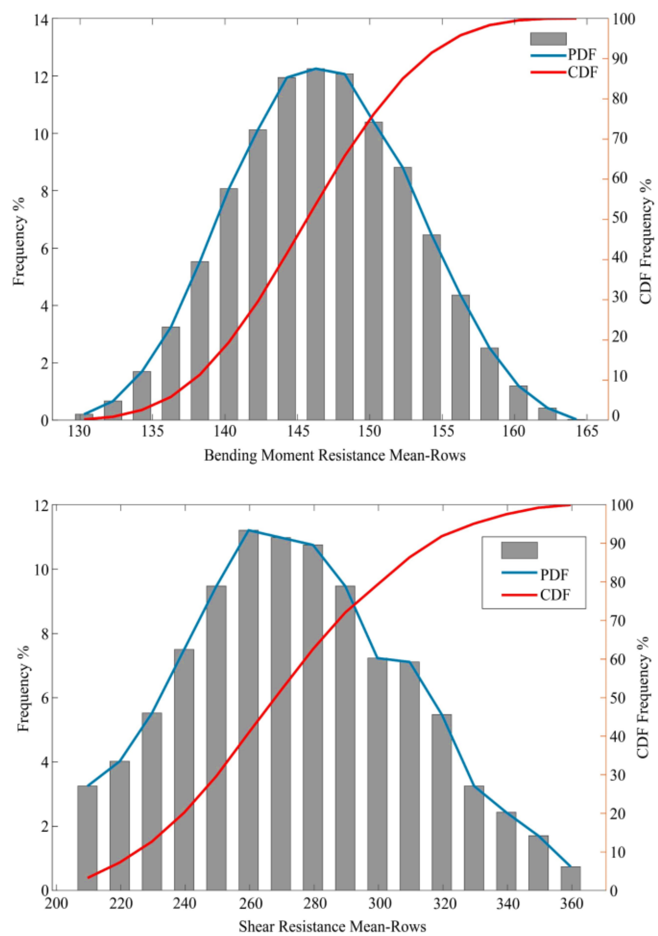

Fig. 4 PDF and CDF of the 50,000 simulations

5.2. Case Study: Reliability Analysis of a RC Bridge

5.2.1. Geometry Properties Variations

The next structure under consideration is a flanged beam of a reinforced concrete bridge. The reliability analysis is conducted by determining the probability of failure due to bending in accordance with EUROCODE 2 [17]. Geometrical property variations are derived from on-site measurements, as shown in Figures 5 to 10. The beam has a fixed span length of 18 meters, and its height ranges from 1.25 to 1.27 meters, with a width varying between 40 and 41 centimeters. These variations in geometry are summarized in Table 9. Figure 11 displays a typical cross-section of the bridge beam.

Table 9. Measurement variation's ranges

Prop.	**Symbol**	**Unit**	**Max**	**Min**
Height	h	m	1.27	1.25
Width	b	cm	41	40
Gusset	g	cm	31	30

Fig. 5 Perspective view of the bridge beams

Fig. 6 Measurement of span length

Fig. 7 Measurement of the height of the beam - 1

Fig. 8 Measurement of the height of the beam - 2

Fig. 9 Measurement of the beam's flange height

Fig. 10 Measurement of the beam's width

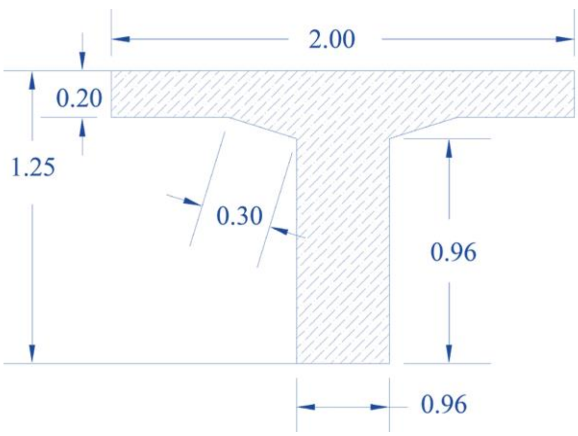

Fig. 11 Typical cross-section of the bridge beam

5.2.2. Limit State Function

As recommended in [20], Concrete compressive strength and reinforcement's yield stress are treated, respectively, as lognormal and normal variables. The design calculation indicates that the section is subjected to a bending moment of 1.4 MN.m due to permanent loads and the superstructure's weight, which includes its own self-weight with a concrete density of 25 kN/m³. Additionally, the bending moment induced by live loads has a value of 3.7 MN.m. Furthermore, the bending moment variables follow a normal distribution [26], [27]. The reinforced concrete design results in a total steel bar area of 150.72 cm², which is treated as a lognormal variable [28]. The verification rule for the ultimate limit state is considered to be violated when the ultimate bending moment exceeds the allowable bending moment resistance. Noting that the concept of failure in the study does not refer to collapse but rather the violation of the design verification criterion. The random variables are outlined in Table 10. Thus, based on [17], the performance function for a flanged beam is described as follows

$$G = \left[0.87A_S f_y\left(d - \frac{h_f}{2}\right) - 0.2bdf_{c28}\frac{(0.36d - h_f)}{2}\right] - (M_{perm} + M_{live}) \quad (10)$$

Table 10. Statistical data of the problem's variables

Var.	Unit	PDF	Nom	Max	Min
f_y	MPa	Log	500	530	470
f_{c28}	MPa	Log	25	29.14	20.86
M_{perm}	MN.m	Norm	1.4	1.512	1.288
M_{live}	MN.m	Norm	3.7	4.07	3.33
h_f	m	Norm	-	1.27	1.25
b	m	Norm	-	0.41	0.40
e	m	Norm	-	0.03	0.02
d	m	Norm	-	1.25	1.22
A_S	cm²	Norm	105.72	121.578	89.862

Table 11. Monte Carlo simulation results for 5000 trials

G(X) Value rows		Mean	PDF	CDF
-2,28	-2,08	-2,18	0,02%	0,02%
-2,08	-1,88	-1,98	0,08%	0,10%
-1,88	-1,68	-1,78	0,18%	0,28%
-1,68	-1,48	-1,58	0,34%	0,62%
-1,48	-1,28	-1,38	1,66%	2,28%
-1,28	-1,08	-1,18	3,54%	5,82%
-1,08	-0,88	-0,98	6,00%	11,82%
-0,88	-0,68	-0,78	9,22%	21,04%
-0,68	-0,48	-0,58	11,84%	32,88%
-0,48	-0,28	-0,38	13,66%	46,54%
-0,28	-0,08	-0,18	14,84%	61,38%
-0,08	0,12	0,02	12,78%	74,16%
0,12	0,32	0,22	10,52%	84,68%
0,32	0,52	0,42	7,30%	91,98%
0,52	0,72	0,62	4,22%	96,20%
0,72	0,92	0,82	2,20%	98,40%
0,92	1,12	1,02	1,02%	99,42%
1,12	1,32	1,22	0,40%	99,82%
1,32	1,52	1,42	0,10%	99,92%
1,52	1,72	1,62	0,04%	99,96%
1,72	1,92	1,82	0,04%	100,00%

Table 12. Probabilities of failure

N	10^2	$5.\ 10^2$	10^3	10^4	10^5	5.10^5	10^6
Pf	0.72	0.59	0.66	0.64	0.62	0.62	0.62

5.2.3. Results

For the presented study, a Monte Carlo simulation is performed to estimate the probability of failure. The resulting probability distribution and cumulative distribution functions, calculated for 5,000 trials, are displayed in the graphs of Figure 12. The estimated values of the limit state function are presented in Table 8. Then, the probability of failure is the value of the cumulative density function for the corresponding red area $G(X) \leq 0$, and it takes a value of 61 %, approximately as illustrated graphically in the same Figure 12. Numerically, the outcome is given by the following equation (11), whereN_F is equal to 3153, and the failure is obtained with a probability of 62%.

$$P_f = \frac{N_F}{N} \quad (11)$$

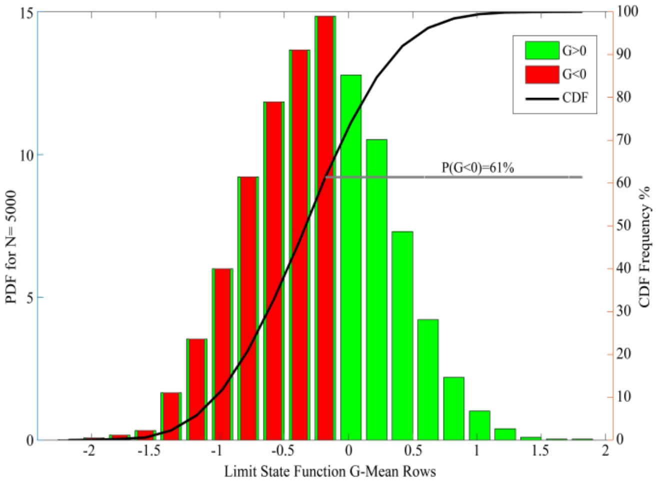

Fig. 12 Statistical distribution of the 5,000 times simulation

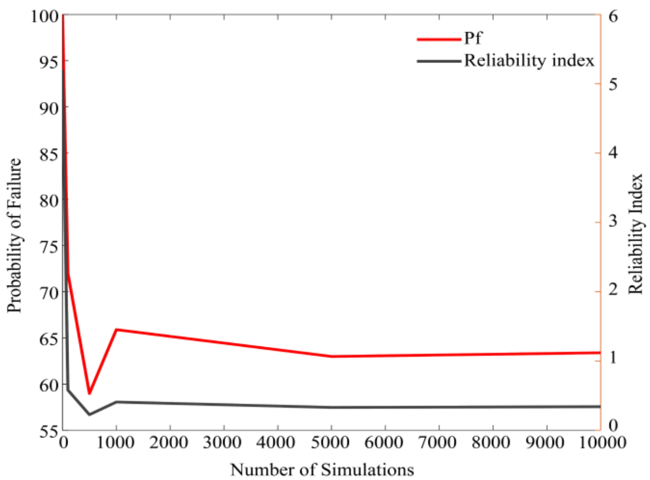

Fig. 13 Convergence of probability of failure and reliability index

To enhance the result's accuracy, the operation was repeated 10^6 times. The finding probability values and the corresponding number of simulations are presented in Table 9. Additionally, the graphs in Figure 13 show the resulting probability of failure and reliability index in line with the number of trials.

6. Discussion on the Potential Benefits and Limitations

The Monte Carlo simulation presents some advantages, as seen in Table 10. As cited in [29], the method is considered an easily comprehensible method without the necessity of a partial derivative as required by approximation methods. Therefore, it allows its application for implicit and complex performance functions. In practice, due to the growing speed of computers, Monte Carlo simulation is often employed to obtain robust approximations to distributions rather than relying on analytical approximation methods [30]. The key advantage of the method is that it is very flexible with parameter distributions, with virtually no limit to the analysis.

Table 13. Advantages of MCS from literature

ADVANTAGES	
The method is considered the most versatile, clear, and easily comprehensible. It does not rely on approximation techniques that involve partial derivatives.	[29]
Good approximations to outcome distributions rather than relying on approximation methods.	[30]
Can generally be easily extended and developed as required and easily understood by no mathematicians.	[31]
FORM and SORM overestimate probabilities of failure, so Monte Carlo simulation is recommended.	[32]
In project management, Monte Carlo simulation is relatively easy to execute and offers insights into the risk associated with investment projects.	[33]

Table 14. Disadvantages of MCS from literature

DRAWBACKS	
The most significant obstacle to Monte Carlo simulation lies in the limited computational power and the considerable time cost for simulation execution.	[4]
If the probability distribution of variables is inappropriate, then the simulation results will also be inadequate.	[4]
The accuracy of solutions is determined by the number of repeated runs performed to generate the output statistics.	[31]

Indeed, the examples have demonstrated some of the advantages of the simulation for estimation purposes and quantification of outcomes, such as probabilistic distribution modeling, statistical sampling of parameters, and evaluation of conditional probabilities. However, using analytical approximation methods such as FORM can become quite challenging, especially when there are a large number of variables and complex performance functions that require multiple iterations for calculation. Therefore, Monte Carlo simulation handles these difficulties with great efficiency. Otherwise, its implementation in this study presents some disadvantages, as given in Table 14. It is a time-consuming process to estimate the output results. The generation of random numbers for a large set of simulations requires a huge amount of sampling, and the convergence of solutions depends on the number of repeated simulations. In addition, referring to [4], the most significant obstacle to the simulation is the insufficiency of computational power and the considerable time required to execute it. Another drawback is the choice of a suitable probability law for each variable. As a result, if the chosen distribution is inappropriate, the simulation results will likewise be inadequate [4]. Hence, data from prior experience and expert knowledge are preferably required. These limitations can be surmounted by integrating them with other approaches, like genetic algorithms, fuzzy logic, and neural networks. In this way, it can extend its applicability across various domains and enhance its effectiveness. Through the merger of Monte Carlo simulation and genetic algorithms, it is possible to conduct structural reliability based on an optimization process. Genetic algorithms, as bio-inspired methods, lead to identifying the most optimal solution for a given problem. The combination of these methods provides powerful tools for assessing and optimizing structures. [34] proposed an optimization algorithm for the reliability design of composite beams that combined Monte Carlo simulation and genetic algorithms.

The method can also be flexibly combined with fuzzy logic, which is a mathematical tool used to deal with uncertainty, decision-making, and imprecision within a system. In reliability analysis, fuzzy logic is utilized to model a system's behavior in the presence of uncertain conditions. [35] developed a fuzzy Monte Carlo method for integrating uncertainty into the planning of green transportation. This approach can help decision-makers in transportation planning to account for uncertainties and make informed decisions based on assessments that are more accurate.

However, the study's findings significantly advance the state of the art in structural reliability and probabilistic fields by providing a viable employment of Monte Carlo simulation for novel aspects that can be considered in reinforced concrete design processes and structural analysis of bridges.

7. Conclusion

The literature review conducted in the paper highlights a variety of applications of Monte Carlo simulation in civil engineering. As a result, the simulation has been demonstrated to be an efficient method for handling complex problems in the field. However, it also presents some drawbacks, such as being time-consuming and requiring a significant amount of computer power.

The practical cases in the study show that the Monte Carlo method is relatively simple to apply and provides significant information regarding strength properties and failure probability. Indeed, it proved to be a significant tool for engineers to incorporate uncertainty into their calculations. Besides, by using the probabilistic approach of the method, engineers can assess the reliability and expect a margin of safety for their projects.

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